



IB MYP YEAR 5

YEAR 10 Mathematics

Assessment #1 POLYNOMIALS

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Teacher: **Ms. Li, Mr. So & Mr. Wong**

Date of task: **Friday, October 5, 2013**

Time allowed: **60 minutes**

Student's Performance in Different Criterion			
A	7	C	6

INSTRUCTIONS:

- ◆ Read the instructions for all questions carefully.
- ◆ Show all work, steps and proper units.
- ◆ Ask the teacher for scrap paper, but any work on the scrap paper will **NOT** be marked.
- ◆ Write in **PENCIL**.
- ◆ **NOT** allowed to use any **electronic devices**, such as translators.
- ◆ Allowed to use **calculators**
- ◆ Allowed to use **non-electronic dictionary**.

ASSESSMENT:

- ◆ Read the criteria descriptors carefully before you start your work. This will give you a clear understanding of what is required and what a quality piece of work for this task must include. This way you give yourself the best chance of achieving the highest level in this task.
- ◆ This task assesses Criteria **A & C**.
 - ✧ For Criteria **A**, the questions are all assigned with levels;
 - ✧ Criterion **C** will be assessed as an **overall impression** on the presentation of work in this assessment.

Criterion A: KNOWLEDGE AND UNDERSTANDING

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	(0-8)
1–2 Simple	The student can solve <u>some</u> simple problems.	The student attempts to make deductions when solving simple problems in familiar contexts.	
3–4 Complex	The student can solve <u>most</u> simple and <u>some</u> more complex problems.	The student sometimes makes appropriate deductions when solving simple and more-complex problems in familiar contexts	Teacher's Final Grade
5–6 Challenging	The student can solve challenging problem correctly and <u>most</u> familiar problems along with <u>all</u> different types of problems.	The student generally makes appropriate deductions when solving challenging problems in a variety of familiar contexts.	(0-8)
7–8 Unfamiliar	The student can solve <u>most</u> challenging and <u>most</u> familiar problems along with <u>all</u> different types of problems.	The student consistently makes appropriate deductions when solving challenging problems in a variety of contexts including unfamiliar situations.	

Criterion C: COMMUNICATION IN MATHEMATICS

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	(0-6)
1–2	The student should be able to explain <u>some problems</u> step by step. The lines of reasoning are <u>difficult to follow</u> .	<ul style="list-style-type: none"> The student shows basic use of mathematical language and/or forms of mathematical representation. The lines of reasoning are difficult to follow. 	
3–4	The student should be able to explain <u>most problems</u> step by step. The lines of reasoning are <u>clear</u> though <u>not always</u> logical or complete .	<ul style="list-style-type: none"> The student shows sufficient use of mathematical language and forms of mathematical representation. The lines of reasoning are clear though not always logical or complete. The student moves between different forms of representation with some success. 	Teacher's Final Grade
5–6	The student should be able to explain <u>most problems</u> step by step. The lines of reasoning are concise, logical and complete . The student use correct unit in the questions.	<ul style="list-style-type: none"> The student shows good use of mathematical language and forms of mathematical representation. The lines of reasoning are concise, logical and complete. The student moves effectively between different forms of representation. 	(0-6)

A. Simple problems

The suggested time allocated for **Question 1 to 4 is 12 minutes.**

1. Solve each of the following equations.

(a) $3(x-2)(x+1) = 0$

$$3(x-2)(x+1) = 0$$

$$\therefore x = 2 \text{ or } x = -1$$

(b) $2(x-1)^2 - 6 = 0$

$$2(x-1)^2 - 6 = 0$$

$$2(x^2 - 2x + 1) - 6 = 0$$

$$2x^2 - 4x + 2 - 6 = 0$$

$$2x^2 - 4x - 4 = 0$$

$$2(x^2 - 2x - 2) = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 + 8}}{2}$$

$$= \frac{2 \pm \sqrt{12}}{2}$$

$$= \frac{2 \pm 2\sqrt{3}}{2}$$

$$\therefore x = 1 \pm \sqrt{3}$$

(c) $3x^2 - 2x - 1 = 0$

$$3x^2 - 2x - 1 = 0$$

$$(3x+1)(x-1) = 0$$

$$\therefore x = -\frac{1}{3} \text{ or } x = 1$$

2. Find the remainder when $f(x) = x^3 + x^2 + 3x + 4$ is divided by $(x-1)$.

$$f(x) = x^3 + x^2 + 3x + 4$$

$$f(1) = 1^3 + 1^2 + 3(1) + 4$$

$$= 1 + 1 + 3 + 4$$

$$= 9$$

\therefore The remainder is 9.

3. If $f(x) = x^4 - 2x^2 + k$ is divisible by $(x+2)$, find k .

$$f(x) = x^4 - 2x^2 + k$$

$$f(-2) = (-2)^4 - 2(-2)^2 + k = 0$$

$$16 - 8 + k = 0$$

$$8 + k = 0$$

$$\therefore k = -8$$

4. If the function of a parabola is $y = 2(x+1)^2 - 5$, find the vertex of the parabola.

By the equation $y = a(x-h)^2 + k$ when (h, k) is the vertex of a parabola,

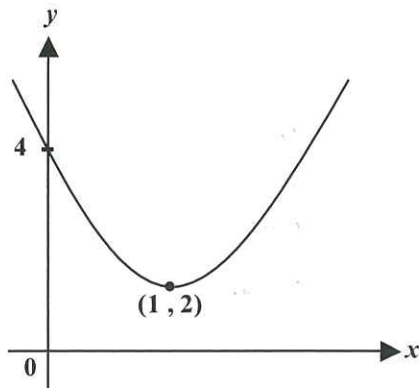
The vertex of the parabola of $y = 2(x+1)^2 - 5$ is $(-1, -5)$

B. More complex problems

The suggested time allocated for **Question 5 to 7 is 18 minutes.**

5. Find the functions of each of the following parabolas.

(a)



where (1, 2) is the **vertex** of the above parabola.

By the equation $y = a(x-h)^2 + k$ where (h, k) is the vertex,

$$y = a(x-1)^2 + 2 \quad \text{--- ①}$$

Sub. the y-intercept (0, 4) into ①

$$\begin{aligned} 4 &= a(0-1)^2 + 2 \\ &= a(-1)^2 + 2 \\ &= a + 2 \end{aligned}$$

$$\begin{aligned} a &= 4 - 2 \\ &= 2 \end{aligned}$$

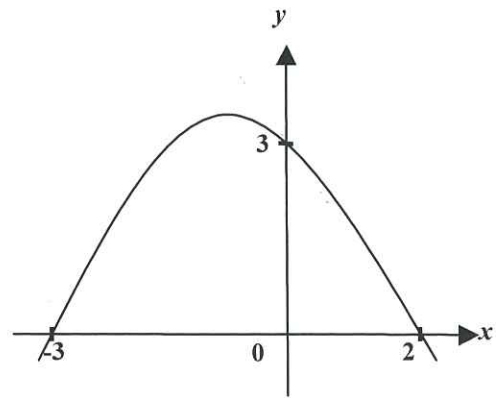
Sub. $a=2$ into ①,

$$y = 2(x-1)^2 + 2$$

\therefore The function of the parabola is

$$y = 2(x-1)^2 + 2$$

(b)



$\therefore (-3, 0)$ and $(2, 0)$ are the roots of the equation,

$$\therefore y = a(x+3)(x-2) \quad \text{--- ①}$$

Sub. the y-intercept (0, 3) into ①,

$$\begin{aligned} 3 &= a(0+3)(0-2) \\ &= 3(-2)a \\ &= -6a \end{aligned}$$

$$a = \frac{3}{-6}$$

$$= -\frac{1}{2}$$

Sub. $a = -\frac{1}{2}$ into ①,

$$y = -\frac{1}{2}(x+3)(x-2)$$

\therefore The function of the parabola is

$$y = -\frac{1}{2}(x+3)(x-2)$$

6. The polynomial $f(x) = x^3 + ax^2 + bx - 3$ is **divisible** by $(x - 3)$. When $f(x)$ is **divided** by $(x + 2)$, the remainder is 15. Find the value of a and b .

$f(x) = x^3 + ax^2 + bx - 3$
 $f(3) = 3^3 + a(3)^2 + b(3) - 3 = 0$
 $27 + 9a + 3b - 3 = 0$
 $9a + 3b + 24 = 0$
 $3(3a + b + 8) = 0$
 $3a + b + 8 = 0$
 $3a + b = -8 \quad \text{--- (1)}$

$f(x) = x^3 + ax^2 + bx - 3$
 $f(-2) = (-2)^3 + a(-2)^2 + b(-2) - 3 = 15$
 $-8 + 4a - 2b - 3 = 15$
 $4a - 2b - 11 = 15$
 $4a - 2b - 11 - 15 = 0$
 $4a - 2b - 26 = 0$
 $2(2a - b - 13) = 0$
 $2a - b - 13 = 0$
 $2a - b = 13 \quad \text{--- (2)}$

$\begin{cases} 3a + b = -8 \quad \text{--- (1)} \\ 2a - b = 13 \quad \text{--- (2)} \end{cases}$

By elimination,
 $(1) + (2): 5a = 5$
 $a = 1$

Sub. $a = 1$ into (1)
 $3(1) + b = -8$
 $3 + b = -8$
 $b = -11$

\therefore The values of a and b are 1 and -11 respectively.

7. If the graph of $y = mx^2 + 12x + 8$ intersects the x-axis and m is a **positive integer**,

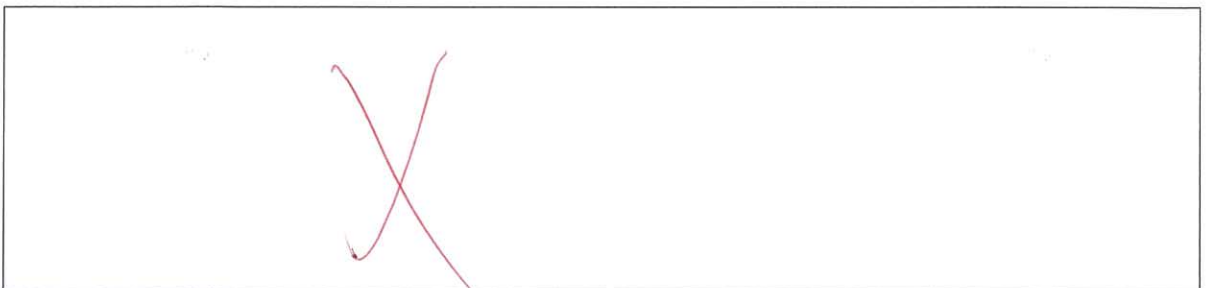
(a) find the FOUR possible values of m .

By cross method of factorisation,

$(mx+4)(x+2) = 0$
 $(mx+2)(x+4) = 0$
 $(mx+1)(x+8) = 0$
 $(mx+8)(x+1) = 0$

\therefore The four possible values of m is $-\frac{4}{x}, -\frac{2}{x}, -\frac{1}{x}, -\frac{8}{x}$.

(b) when m is a **maximum**, find the **roots** of the equation $mx^2 + 12x + 8 = 0$.



C. Challenging problem

The suggested time allocated for **Question 8** is 15 minutes.

8. Given that the equation $(2k - 1)x^2 + (k - 3)x - 2 = 0$ where $k \neq \frac{1}{2}$,

- (a) find the value of k if the **sum of roots equals the product of roots** of the equation.

$$(2k-1)x^2 + (k-3)x - 2 = 0$$

$$\alpha + \beta = -\frac{k-3}{2k-1}$$

$$\alpha\beta = \frac{-2}{2k-1}$$

Given sum of roots = product of roots,

$$-\frac{k-3}{2k-1} = \frac{-2}{2k-1}$$

$$-\frac{k-3}{2k-1} - \frac{-2}{2k-1} = 0$$

$$\frac{-(k-3) - (-2)}{2k-1} = 0$$

$$\frac{-k+3+2}{2k-1} = 0$$

$$-k+5 = 0$$

$$(2k-1)^2 \left(\frac{-k+5}{2k-1} \right) = 0 (2k-1)^2$$

$$(2k-1)(-k+5) = 0$$

$$-(2k-1)(k-5) = 0$$

$$\therefore k = \frac{1}{2} \text{ or } k = 5$$

$\therefore k \neq \frac{1}{2}$
 $k = \frac{1}{2}$ is invalid
 \therefore The value of k is 5.

- (b) By using the value of k obtained in (a), find the **axis of symmetry**, **x-intercept(s)** and **y-intercept**.

Sub. $k=5$ into $(2k-1)x^2 + (k-3)x - 2 = 0$

$$(2 \cdot 5 - 1)x^2 + (5 - 3)x - 2 = 0$$

$$9x^2 + 2x - 2 = 0$$

Axis of symmetry: $\left(-\frac{b}{2a}, \frac{4ac - b^2}{2a} \right)$

$$= \left(-\frac{2}{2(9)}, \frac{4(9)(-2) - (2)^2}{2(9)} \right)$$

$$= \left(-\frac{2}{18}, \frac{-72 - 4}{18} \right)$$

$$= \left(-\frac{1}{9}, -\frac{38}{9} \right)$$

x-intercept(s): $x = \frac{-2 \pm \sqrt{2^2 - 4(9)(-2)}}{2(9)}$

$$= \frac{-2 \pm \sqrt{4 + 72}}{18}$$

$$= \frac{-2 \pm \sqrt{76}}{18}$$

$$= \frac{-2 \pm 2\sqrt{19}}{18}$$

$$= \frac{-1 \pm \sqrt{19}}{9}$$

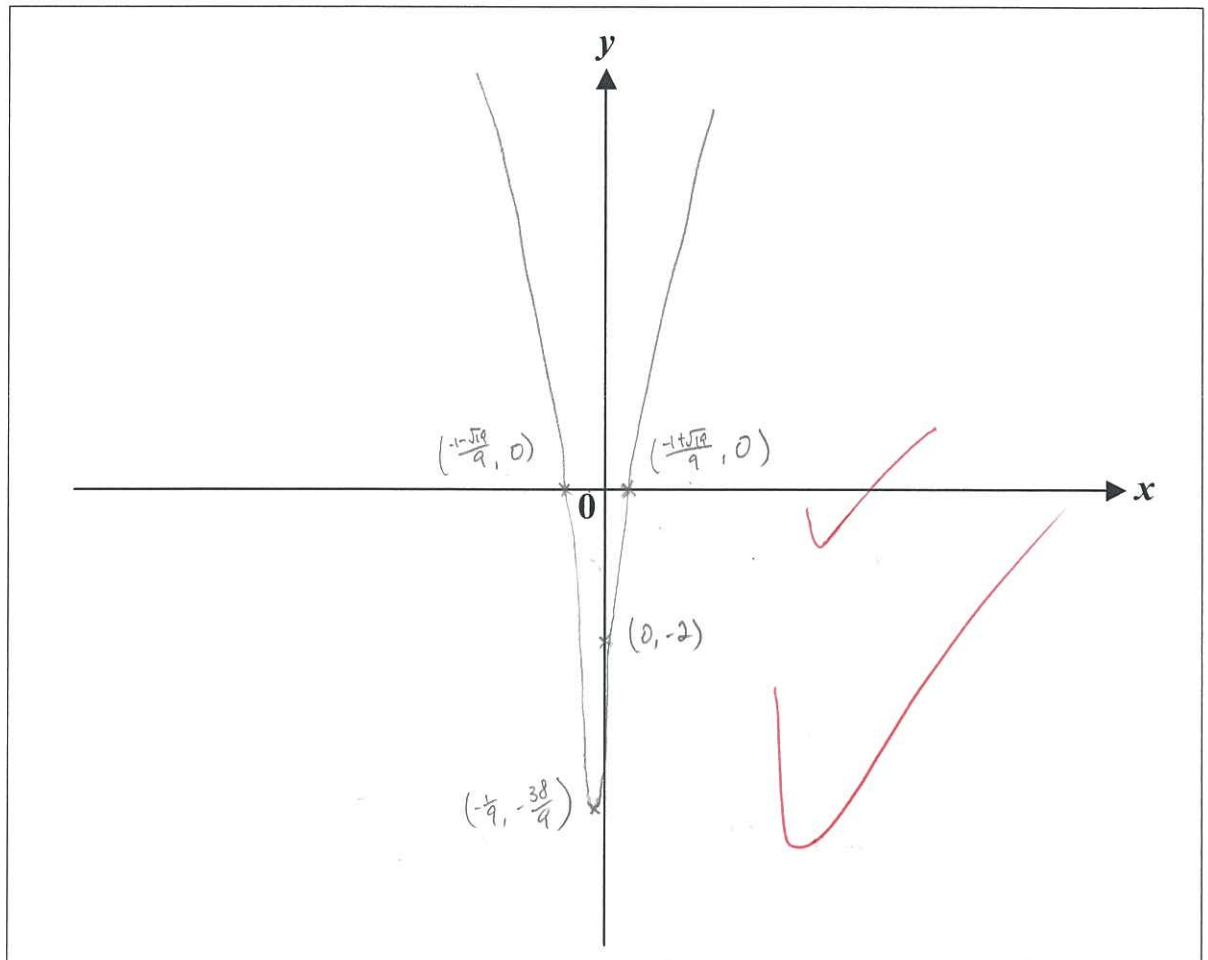
\therefore The x-intercepts are $\left(\frac{-1 + \sqrt{19}}{9}, 0 \right)$ and $\left(\frac{-1 - \sqrt{19}}{9}, 0 \right)$

y-intercept: By the equation $y = ax^2 + bx + c$ where $(0, c)$ is the y-intercept,

$\therefore c = -2$

\therefore The y-intercept is $(0, -2)$

- (c) By using the results **obtained in (b)**, sketch the function $y = (2k - 1)x^2 + (k - 3)x - 2$. The **axis of symmetry**, **x-intercept(s)** and **y-intercept** should be clearly shown on your graph.



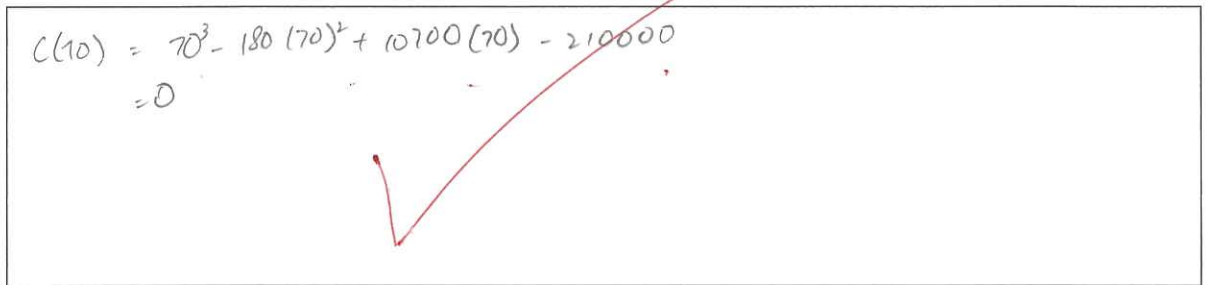
D. Unfamiliar problems

The suggested time allocated for **Question 9 is 15 minutes**.

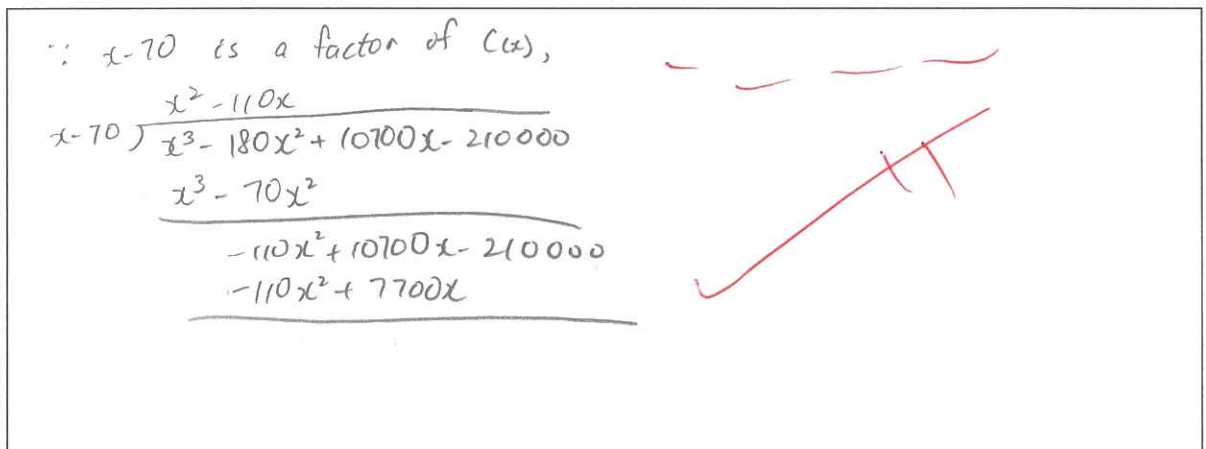
9. A courier company is responsible for delivering documents to Mainland China. Suppose the volume (in cm^3) of the rectangular carton used for delivery is given by $C(x) = x^3 - 180x^2 + 10700x - 210000$ (where $x > 70$).

MTR is the main means of transport used by that company. In order to reduce the cost, the cartons used must conform to the restrictions on the size of luggage carried by passengers on the MTR: **the sum of the length, width and height of the luggage should not exceed 170 cm, and the length of any side of the luggage should not exceed 130 cm.**

- (a) Show that $x - 70$ is a factor of $C(x)$.

$$\begin{aligned} C(70) &= 70^3 - 180(70)^2 + 10700(70) - 210000 \\ &= 0 \end{aligned}$$


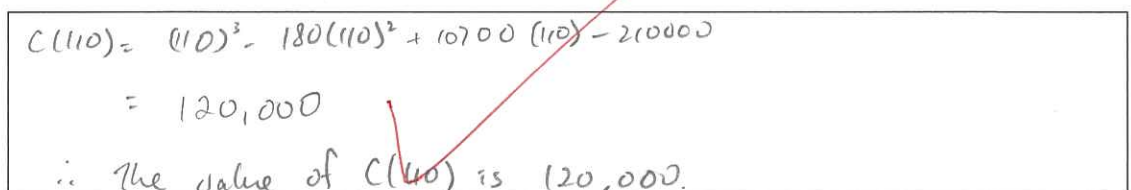
- (b) Factorize $C(x)$.

$$\begin{array}{r} \because x-70 \text{ is a factor of } C(x), \\ x^2 - 110x \\ x-70 \overline{) x^3 - 180x^2 + 10700x - 210000} \\ \underline{x^3 - 70x^2} \\ -110x^2 + 10700x - 210000 \\ \underline{-110x^2 + 7700x} \\ -3000x - 210000 \end{array}$$


- (c) (i) Find the value of $C(110)$.

$$\begin{aligned} C(110) &= (110)^3 - 180(110)^2 + 10700(110) - 210000 \\ &= 120,000 \end{aligned}$$

\therefore The value of $C(110)$ is 120,000.



- (ii) Using the results of (b) and (c)(i), suggest the dimensions of a carton, with volume 120000cm^3 , which conform to the restrictions on the size of luggage carried by passengers on the MTR.



End of Assessment